Erratum: Universal back-projection algorithm for photoacoustic computed tomography [Phys. Rev. E 71, 016706 (2005)]

Minghua Xu* and Lihong V. Wang†

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We would like to point out the errors in the proof of $P^{(2)}(\mathbf{r}',\mathbf{r})=0$ in this paper. These errors, however, do not affect the results of the paper. For completeness, we would like to rewrite the proof from Eq. (12) to Eq. (18) on p. 016706-3. Starting with Eq. (12) of the paper, the second term can be rewritten as

$$P^{(2)}(\mathbf{r}',\mathbf{r}) = \frac{1}{2\pi}[\varepsilon^+ + (\varepsilon^+)^*], \tag{12}$$

where the asterisk denotes complex conjugation and $\varepsilon^+ = i\int_0^{+\infty}\tilde{F}_k(\mathbf{r}',\mathbf{r})kdk$ with

$$\tilde{F}_k(\mathbf{r}',\mathbf{r}) = \int_{V_0} dV_0(\boldsymbol{\nabla}+\boldsymbol{\nabla}')^2[\tilde{G}_k^{(out)}(\mathbf{r}',\mathbf{r}_0)\tilde{G}_k^{(in)}(\mathbf{r},\mathbf{r}_0)].$$

As defined in the paper, $\boldsymbol{\nabla}_0$, $\boldsymbol{\nabla}'$, and $\boldsymbol{\nabla}$ denote gradients over the variables $\mathbf{r}_0$, $\mathbf{r}'$, and $\mathbf{r}$, respectively. It is easy to show, $\boldsymbol{\nabla}_0|\mathbf{r}'-\mathbf{r}_0| = -\boldsymbol{\nabla}'|\mathbf{r}'-\mathbf{r}_0|$ and $\boldsymbol{\nabla}_0|\mathbf{r}-\mathbf{r}_0| = -\boldsymbol{\nabla}|\mathbf{r}-\mathbf{r}_0|$. Hence, we have

$$(\boldsymbol{\nabla}+\boldsymbol{\nabla}')^2[\tilde{G}_k^{(out)}(\mathbf{r}',\mathbf{r}_0)\tilde{G}_k^{(in)}(\mathbf{r},\mathbf{r}_0)] = \boldsymbol{\nabla}_0\cdot\boldsymbol{\nabla}_0[\tilde{G}_k^{(out)}(\mathbf{r}',\mathbf{r}_0)\tilde{G}_k^{(in)}(\mathbf{r},\mathbf{r}_0)].$$

Therefore, $\tilde{F}_k(\mathbf{r}',\mathbf{r})$ can be rewritten as a surface integral based on Gauss's theorem,

$$\tilde{F}_k(\mathbf{r}',\mathbf{r}) = \int_{V_0} dV_0\boldsymbol{\nabla}_0\cdot\boldsymbol{\nabla}_0[\tilde{G}_k^{(out)}(\mathbf{r}',\mathbf{r}_0)\tilde{G}_k^{(in)}(\mathbf{r},\mathbf{r}_0)] = \int_S dS(-\mathbf{n}_0^S)\cdot\boldsymbol{\nabla}_0[\tilde{G}_k^{(out)}(\mathbf{r}',\mathbf{r}_0)\tilde{G}_k^{(in)}(\mathbf{r},\mathbf{r}_0)]. \tag{13}$$

It has to be pointed out that here we must treat $\mathbf{r}_0$ as a free variable when performing $\boldsymbol{\nabla}_0$, and then fix $\mathbf{r}_0$ on surface S for surface integral. In the planar geometry, we obtain [1] ($\Delta z > 0$)

$$\tilde{G}_k^{(in)}(\mathbf{r},\mathbf{r}_0) = \frac{1}{(2\pi)^3}\int\int_{-\infty}^{+\infty} du\, dv\, \exp(-iu\Delta x - iv\Delta y)\left\{-\chi\left(\frac{\rho}{k}\right)i\pi\,\mathrm{sgn}(k)\frac{\exp[-i\Delta z\,\mathrm{sgn}(k)w]}{w} - \chi\left(\frac{k}{\rho}\right)\pi\frac{\exp(-\Delta z w)}{w}\right\} \tag{14}$$

and $\tilde{G}_k^{(out)}(\mathbf{r}',\mathbf{r}_0) = [\tilde{G}_k^{(in)}(\mathbf{r}',\mathbf{r}_0)]^*$ by replacing $\mathbf{r}=(x,y,z)$ with $\mathbf{r}'=(x',y',z')$, ρ with ρ', and (u,v,w) with (u',v',w'), where $\Delta x = x - x_0$, etc. Furthermore, $\chi(\xi)=1$ for $|\xi|<1$ and 0, otherwise, $\mathrm{sgn}(k)=1$ for $k>0$ and -1 for $k<0$, $\rho=\sqrt{u^2+v^2}$, and $w=\sqrt{|k^2-\rho^2|}$. Here, we assume that the source is above the measurement plane $z_0=0$. Because $dS=dx_0dy_0$ and $-\mathbf{n}_0^S\cdot\boldsymbol{\nabla}_0 = -\partial/\partial z_0$ ($\mathbf{n}_0^S$ is along $\mathbf{z}_0$), substituting $\tilde{G}_k^{(out)}(\mathbf{r}',\mathbf{r}_0) = [\tilde{G}_k^{(in)}(\mathbf{r}',\mathbf{r}_0)]^*$ and Eq. (14) into (13) gives

$$\begin{aligned}\tilde{F}_k(\mathbf{r}',\mathbf{r}) &= -\int_S dS\frac{\partial}{\partial z_0}[\tilde{G}_k^{(out)}(\mathbf{r}',\mathbf{r}_0)\tilde{G}_k^{(in)}(\mathbf{r},\mathbf{r}_0)] = -\frac{\partial}{\partial z_0}\int_{x_0} dx_0\int_{y_0} dy_0\tilde{G}_k^{(out)}(\mathbf{r}',\mathbf{r}_0)\tilde{G}_k^{(in)}(\mathbf{r},\mathbf{r}_0)\\ &= \frac{-1}{8\pi^2}\int\int du\, dv\, \exp[-iu(x-x')-iv(y-y')]\chi\left(\frac{k}{\rho}\right)\frac{\exp[-(z'+z)w]}{w}.\end{aligned}$$

It is easy to show $\tilde{F}_k(\mathbf{r}',\mathbf{r})^* = \tilde{F}_k(\mathbf{r}',\mathbf{r})$. Therefore, $\tilde{F}_k(\mathbf{r}',\mathbf{r})$ is real.

In the spherical geometry, we obtain ($k>0$) [2]

*Present address: Abramson Family Cancer Research Institute at the University of Pennsylvania School of Medicine, Room 627 BRB II/III, 421 Curie Boulevard, Philadelphia, PA 19104-6160.

†Author to whom correspondence should be addressed. Also at the Department of Biomedical Engineering at Washington University in St. Louis, One Brookings Drive, Campus Box 1097, St. Louis, MO 63130-4899. FAX: (314) 935-7448. Electronic mail: lhwang@biomed.wustl.edu; URL: http://oilab.seas.wustl.edu

$$\tilde{G}_k^{(in)}(\mathbf{r},\mathbf{r}_0) = \frac{-ik}{4\pi}\sum_{l=0}^{\infty}(2l+1)j_l(kr)h_l^{(2)}(kr_0)P_l(\mathbf{n}\cdot\mathbf{n}_0), \tag{15}$$

and $\tilde{G}_k^{(out)}(\mathbf{r}',\mathbf{r}_0)=[\tilde{G}_k^{(in)}(\mathbf{r}',\mathbf{r}_0)]^*$ by replacing $\mathbf{n}$ by $\mathbf{n}'$, where $j_l(\cdot)$ is the spherical Bessel function of the first kind, $h_l^{(2)}(\cdot)$ is the spherical Hankel function of the second kind, and $P_l(\cdot)$ is the Legendre polynomial. Furthermore, $\mathbf{n}'=\mathbf{r}'/r$, $\mathbf{n}_0=\mathbf{r}_0/r_0$, and $\mathbf{n}=\mathbf{r}/r$. Because $dS=r_0^2 d\Omega_0$ and $-\mathbf{n}_0^S\cdot\nabla_0=\partial/\partial r_0$ ($\mathbf{n}_0^S$ is along $-\mathbf{r}_0$), we rewrite Eq. (13) as

$$\tilde{F}_k(\mathbf{r}',\mathbf{r}) = \int_S dS\frac{\partial}{\partial r_0}[\tilde{G}_k^{(out)}(\mathbf{r}',\mathbf{r}_0)\tilde{G}_k^{(in)}(\mathbf{r},\mathbf{r}_0)] = r_0^2\frac{\partial}{\partial r_0}\int_{\Omega_0} d\Omega_0\tilde{G}_k^{(out)}(\mathbf{r}',\mathbf{r}_0)\tilde{G}_k^{(in)}(\mathbf{r},\mathbf{r}_0).$$

Then, substituting $\tilde{G}_k^{(out)}(\mathbf{r}',\mathbf{r}_0)=[\tilde{G}_k^{(in)}(\mathbf{r}',\mathbf{r}_0)]^*$ and Eq. (15) into the above equation gives

$$\tilde{F}_k(\mathbf{r}',\mathbf{r}) = \frac{k^2}{4\pi}\sum_{l=0}^{\infty}(2l+1)j_l(kr)j_l(kr')P_l(\mathbf{n}\cdot\mathbf{n}')r_0^2\frac{\partial}{\partial r_0}m_l^2(kr_0). \tag{16}$$

Here, $m_l^2(kr_0)=j_l^2(kr_0)+n_l^2(kr_0)$, where $n_l(\cdot)$ denotes the spherical Bessel function of the second kind. Therefore, $\tilde{F}_k(\mathbf{r}',\mathbf{r})$ is real.

In cylindrical geometry, we denote $\mathbf{r}'=(\rho',\varphi',z')$, $\mathbf{r}=(\rho,\varphi,z)$, and $\mathbf{r}_0=(\rho_0,\varphi_0,z_0)$. In this case, we obtain ($k>0$) [1,3]

$$\tilde{G}_k^{(in)}(\mathbf{r},\mathbf{r}_0) = \frac{1}{4\pi^2}\sum_{n=-\infty}^{+\infty}\exp[in(\varphi_0-\varphi)]\int_{-\infty}^{+\infty}dk_z\exp[ik_z(z_0-z)]\left[\frac{-i\pi}{2}\chi\left(\frac{k_z}{k}\right)J_n(\mu\rho)H_n^{(2)}(\mu\rho_0)+\chi\left(\frac{k}{k_z}\right)I_n(\mu\rho)K_n(\mu\rho_0)\right], \tag{17}$$

and $\tilde{G}_k^{(out)}(\mathbf{r}',\mathbf{r}_0)=[\tilde{G}_k^{(in)}(\mathbf{r}',\mathbf{r}_0)]^*$ by replacing n with n', k_z with k_z', and μ with μ', respectively, where $\mu=\sqrt{|k^2-k_z^2|}$ and $\mu'=\sqrt{|k^2-k_z'^2|}$, $J_n(\cdot)$ is the Bessel function of the first kind, $H_n^{(2)}(\cdot)$ is the Hankel function of the second kind, $I_n(\cdot)$ is the modified Bessel function of the first kind, and $K_n(\cdot)$ is the modified Bessel function of the second kind. Because $dS=\rho_0 d\varphi_0 dz_0$ and $-\mathbf{n}_0^S\cdot\nabla_0=\partial/\partial\rho_0$ ($\mathbf{n}_0^S$ is along $-\boldsymbol{\rho}_0$), we rewrite Eq. (13) as

$$\tilde{F}_k(\mathbf{r}',\mathbf{r}) = \int_S dS\frac{\partial}{\partial\rho_0}[\tilde{G}_k^{(out)}(\mathbf{r}',\mathbf{r}_0)\tilde{G}_k^{(in)}(\mathbf{r},\mathbf{r}_0)] = \rho_0\frac{\partial}{\partial\rho_0}\int_{\varphi_0}d\varphi_0\int_{z_0}dz_0\tilde{G}_k^{(out)}(\mathbf{r}',\mathbf{r}_0)\tilde{G}_k^{(in)}(\mathbf{r},\mathbf{r}_0).$$

Then, substituting $\tilde{G}_k^{(out)}(\mathbf{r}',\mathbf{r}_0)=[\tilde{G}_k^{(in)}(\mathbf{r}',\mathbf{r}_0)]^*$ and Eq. (17) into the above equation gives

$$\begin{aligned}\tilde{F}_k(\mathbf{r}',\mathbf{r}) = {} & \frac{1}{4\pi^2}\rho_0\sum_{n=-\infty}^{+\infty}\exp[in(\varphi'-\varphi)]\int_{-\infty}^{+\infty}dk_z\exp[ik_z(z'-z)] \\ & \times\left[\frac{\pi^2}{4}\chi\left(\frac{k_z}{k}\right)J_n(\mu\rho')J_n(\mu\rho)\frac{\partial}{\partial\rho_0}M_n^2(\mu\rho_0)+\chi\left(\frac{k}{k_z}\right)I_n(\mu\rho')I_n(\mu\rho)\frac{\partial}{\partial\rho_0}K_n^2(\mu\rho_0)\right].\end{aligned} \tag{18}$$

Here, $M_n^2(\mu\rho_0)=J_n^2(\mu\rho_0)+N_n^2(\mu\rho_0)$, where $N_n(\cdot)$ is the Bessel function of the second kind. It is easy to show $\tilde{F}_k(\mathbf{r}',\mathbf{r})^*=\tilde{F}_k(\mathbf{r}',\mathbf{r})$. Therefore, $\tilde{F}_k(\mathbf{r}',\mathbf{r})$ is real.

In summary, $\tilde{F}_k(\mathbf{r}',\mathbf{r})$ is real for all three geometries. Hence, ε^+ is purely imaginary. From Eq. (12), $P^{(2)}(\mathbf{r}',\mathbf{r})=0$.

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